ARITHMETIC PROPERTIES OF SOME ROTATIONS AND INTERVAL MAPS VIA LINEAR "RECURRENCIES"

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ABSTRACT. The linear map $\begin{bmatrix} 0 & -1 \\ 1 & t \end{bmatrix}$ for a rational $-2 < t < 2$ is an irrational rotation on the invariant ellipses. Any rational orbit is also a linear "recurrence". Traditionally number theorists studied only integer sequences, thus excluding this scenario.

For the interval map $\Psi(x) = x^2 - 2$ on $[-2, 2]$ rational orbits are also associated with linear "recurrencies", but in a different way.

We will present a new way to study the linear recursive sequences of order two. It is based on attaching a ring structure and an abelian group structure to the family of sequences for one set of parameters and different initial conditions. The elements of the *sequence group* are sequences identified up to scalar multiplication. The group is similar, but not identical to the Laxton group.

These structures are compared for different sets of parameters. In particular, the traditional two integer parameters are replaced with one rational parameter (not so surprising if you recall the magic of rational numbers).

One of the highlights is the discovery that the Fibonacci sequence

$$x_{n+1} = x_n + x_{n-1}, x_0 = 0, x_1 = 1,$$

has a "twin"

$$y_{n+1} = 5y_n - 5y_{n-1}, y_0 = 0, y_1 = 1.$$

Their connection is that the even numbered elements of the two sequences coincide (after the powers of 5 are factored out). The odd numbered elements have disjoint sets of prime divisors, both of prime density 1/3.

The talk is based on the papers "On sequence groups" (arXiv:2110.00450) (joint with Z. Lipiński) and "Partitions of primes by Chebyshev polynomials" (arXiv:1806.09446).

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