A "dynamic" method to study the monochromatic arithmetic progressions in the Fibonacci word

Gandhar Joshi

gandhar.joshi@open.ac.uk

The Open University

(This talk is based on joint work with Dan Rust.)

Take a sequence with each n-th term as a function of natural numbers n (including 0) onto a finite alphabet. the Fibonacci word $(f_n)_{n\geq 0}$ =0100101001001101... is one such example (a.k.a. the Rabbit sequence). Here, the output of the function (f_n) is the parity of the Zeckendorf representation of n (a.k.a. least significant digit). For example,

$$f_{12} = \text{ end term of } (12)_Z = 10101 = 1.$$

A (finite or infinite) monochromatic arithmetic progression (MAP) with difference d in a sequence of symbols is counting the appearance of the same symbol at positions with a MAP difference d. For example, in the Fibonacci word above, for $d = 3$, $f_2 = f_5 = f_8 = f_{11} = 0$. Durand and Goyheneche showed that the Fibonacci word does not admit infinite MAPs. We prove an elegant formulation of the longest MAP length $A(d)$ where d is a Fibonacci number ($F_{n\geq 2} = 1, 2, 3, 5, 8, \dots$).

Proposition 1. *For all $n \geq 2$, $A(F_n) = \lceil \tau^{n-1} \rceil$, where τ is the golden ratio $1.618\dots$.*

We use a definition of the Fibonacci word that comes from the topic of dynamical systems to prove Proposition 1, and ask further questions arising from the complexity in this method.