

Title: Geometry and distribution of roots of quadratic congruences

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Abstract:

The recent paper by Marklof and Welsh established limit laws for the fine-scale distribution of the roots of the quadratic congruence $\mu^2 \equiv D \pmod{m}$, ordered by the modulus m , where D is a square-free positive integer and $D \not\equiv 1 \pmod{4}$. This is achieved by relating the roots (when $D > 0$) to the tops of certain geodesics in the Poincaré upper half-plane and (when $D < 0$) to orbits of points, under the action of the modular group.

In this talk, we will investigate the remaining case when $D > 1$ is a square-free integer and $D \equiv 1 \pmod{4}$. We will understand how the roots can be related to the tops of the geodesics by considering ideals of the ring $\mathbb{Z}[\sqrt{D}]$ and the ring of integers of the quadratic number field $\mathbb{Q}(\sqrt{D})$. As two simple examples, the graphs of pair correlations of $D = 5$ and $D = 17$ will be displayed. This is joint work with Matthew Welsh.