The celebrated Birkhoff ergodic theorem asserts that in an ergodic system orbits of almost every points equidistributes when sampled at integer time. This result was generalized by Bourgain (and others) to many natural sparse subsets of the integers. On the other hand our understanding of the behaviour of orbits of **all** points in a dynamical system is much less understood, especially for sparse subsets of the integers. We generalise a method introduced by A. Venkatesh to tackle this problem in two directions, general $\mathbb{R}^d$ actions instead of flows, and weak mixing, rather than mixing, actions. On the way we also show some basic properties of weak mixing, and show weak mixing for the time 1-map of a weak mixing flow.
The main examples are equidistribution along $\{(n^{1+\epsilon_i})_{i=1}^d\}$, which is a direct generalisation of Venkatesh's example, and random subsets of $\mathbb{Z}^d$ for horosperical action.